Fermi National Accelerator Laboratory

UM-TH-93-04

FERMILAB-Pub-93/025-A
February 1993

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submitted to Physical Review Letters
1 February, 1993

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(submitted to Phys. Rev. Lett., February 1, 1993)

Two new extensions of the Parker bound on the flux $\mathcal{F}$ of magnetic monopoles lead to stronger bounds than obtained previously: 1) survival and growth of a small Galactic seed field requires $\mathcal{F} \leq 10^{-16}(m/10^{17}\text{GeV})$ cm^{-2} s^{-1} sr^{-1}, and 2) survival of a seed field through protogalactic collapse places even more stringent limits, but with greater uncertainties.

14.80.Hv, 95.30Cq, 96.40.-z, 98.60Jk, 98.80.-k

The possibility that magnetic monopoles may exist in the universe has long intrigued both theorists and experimentalists. Dirac [1] first showed that magnetic monopoles could be accommodated within electromagnetic theory if their magnetic charge, g, is given by an integer multiple of $\hbar c/2e$. 't Hooft and Polyakov [2] later showed that magnetic monopoles arise as topological defects in gauge theories; in particular, monopoles are a generic feature of Grand Unified Theories (GUTs). The mass of GUT monopoles is usually set by the scale of unification, thought to be $\sim 10^{15}$ GeV or so; $m \sim m_{GUT}/\alpha_{GUT} \sim 10^{17}$ GeV [3].

In the standard cosmology, the number of magnetic monopoles produced in a GUT phase transition is far too large to be compatible with the observed energy density of the universe: the "monopole problem" [3]. In inflationary models, massive entropy production reduces the monopole abundance within the obser vable universe to an exponentially small value [4]. At present, no clear prediction exists for the expected density of monopoles in the universe. Astrophysics, however, can provide clues for experimentalists about what monopole flux to expect.

In the last ten years the experimental search for GUT monopoles has intensified. Their large masses ($\sim 10^{17}$GeV) and small velocities ($v \sim 10^{-3}c$) alerted experimentalists to the possibility that conventional detection techniques may have missed the monopoles. Initially, small induction experiments were tried [5]. However, once it was shown that scintillators could respond to slow GUT monopoles [6], detectors were built with sensitivities to fluxes approaching the astrophysical bounds. With the largest such detector coming on line [7], it is appropriate to reconsider these bounds.

Astrophysical bounds on the magnetic monopole flux fall into three classes: 1) bounds based on the mass density of monopoles either locally or in the universe, 2) bounds based on monopole catalysis of nucleon decay in neutron stars and white dwarfs, 3) bounds based on monopoles draining energy from astrophysical magnetic fields. While flux limits based upon monopole catalysis of nucleon decay are the most stringent [8], it is not obligatory that monopoles catalyze nucleon decay.

The original Parker bound, $\mathcal{F} \leq 10^{-16}$ cm^{-2} s^{-1} sr^{-1}, was obtained by requiring survival of today's magnetic field in the Galaxy, $B \sim 3 \times 10^{-6}$ G [9]. This bound was reexamined and shown to be mass dependent [10]. In this Letter, we strengthen the Parker bound by considering the evolution of a much smaller seed field early in the history of our Galaxy, $B \sim 10^{-20} - 10^{-11}$ G. This smaller field also had to survive the flux of monopoles traveling through the Galaxy. We obtain two bounds: a) survival of the field from the time of the formation of the Galaxy to today, and b) survival of the field during protogalactic collapse. The second bound is stronger but more speculative.

Bounds from Evolution of the Magnetic Field in the Galaxy: The time evolution of the magnetic field in the Galaxy is determined by competition between dynamo action, turbulent dissipation, and (possible) dissipation by a flux of magnetic monopoles. Although the details can be quite complicated [9,11,12], we obtain a good estimate of the behavior of the magnetic field strength B through the equation

$$\frac{dB}{dt} = \gamma B - \alpha B^2 - \frac{Fg}{1 + \mu/B}, \tag{1}$$

where all quantities have been written in dimensionless form: B is the magnetic field strength in units of 3×10^{-6} G (the present day Galactic field strength); γ is the growth rate of the field due to the Galactic dynamo in units of $10^{-8}\mathrm{yr}^{-1}$ (the Galactic rotation rate); t is time in units of 10^{8} yr; α represents the action of turbulent dissipation in units of $(300\ \text{G-yr})^{-1}$. The final term represents the dissipation of the magnetic field due to a flux of magnetic monopoles: here F is the flux in units of $1.2 \times 10^{-16}\mathrm{cm}^{-2}\mathrm{sr}^{-1}\mathrm{s}^{-1}$ and g is the magnetic charge in Dirac units (we take g=1). There is a net kinetic energy gained by monopoles passing through the Galaxy resulting in a net drain of energy from the magnetic field [9]. The particular form for the dissipation term depends on the quantity $\mu \equiv m_{17} v^2/\ell g$ where m_{17} is the mass of the monopole in units of 10^{17} GeV, ℓ is the coherence length of the magnetic field in units of 1 kpc, and v is the velocity of the monopoles as they impinge upon the magnetic region of the Galaxy, in units of $10^{-3}c$ [10]. We expect a massive monopole to acquire this velocity due to gravitational acceleration by the Galaxy during infall. Light monopoles ($\mu \ll$ B) will be accelerated to higher velocities by the Galactic field, while "heavy monopoles" ($\mu \gg$ B) do not have their velocities changed significantly. At our position immersed in the magnetic region of the Galaxy, we expect light monopoles to be moving much faster than $10^{-3}c$, while heavy monopoles should be moving at about $10^{-3}c$.

By defining

$$V(B) = \frac{1}{3}\alpha B^3 - \frac{1}{2}\gamma B^2 + FB - F\mu \ln[\mu + B], \qquad (2)$$

we rewrite Eq. (1) as

$$\frac{dB}{dt} = -\frac{dV}{dB}, \qquad (3)$$

where we have implicitly assumed that the parameters γ, α, and μ all vary sufficiently slowly that they can be taken to be constants.

The behavior of B(t) is determined by V: The extrema of V are the fixed points for the evolution of B; further, maxima are unstable fixed points and minima are stable fixed points. If the monopole flux exceeds the critical value $F_c = (\mu\alpha + \gamma)^2/4\alpha$, then V has a single extremum, a minimum at B = 0. In this case, the field strength evolves toward zero for all initial conditions. Thus $F > F_c$ is not allowed. In the opposite limit, $F < F_c$, the potential has three extrema at B = 0, B_+, and B_-, where B_+ is of order the present strength of the Galactic field. Two possibilities for the shape of the potential exist: 1) if

$$F < \gamma\mu, \qquad (4)$$

then V has a maximum at B=0 and the field will evolve toward the minimum at $B = B_+$ (see Fig. 1a). Thus, condition (4) represents a sufficient (but not necessary) condition on the monopole flux for the survival of the Galactic field. 2) If $F > \gamma\mu$, the potential has minima at both B = 0 and $B = B_+$ and a maximum at $B_- > 0$ provided that $\gamma > \mu\alpha$ (see Fig. 1b). In this case, the field evolves to the value B_+ provided that the initial seed field strength B_0 is sufficiently large,

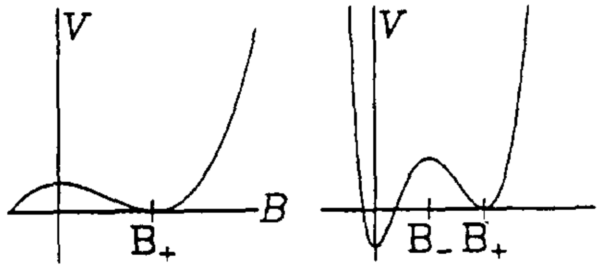

FIG. 1. Sketch of V for $F < (\gamma + \mu\alpha)^2/4\alpha$. (a) For monopole flux $F < \mu\gamma$, there exists a single minimum (for positive field strength) at B_+. (b) For monopole flux $F > \mu\gamma$, there exist two minima ($B = 0, B_+$). In this case, initial field strengths of $B_0 > B_-$ are required to ensure evolution to $B = B_+$.

$$B_0 > B_- = \frac{1}{2\alpha}\{(\gamma - \mu\alpha) - \left[(\gamma - \mu\alpha)^2 - 4\alpha(F - \gamma\mu)\right]^{\frac{1}{2}}\}. \qquad (5)$$

This latter condition implies a flux limit of the form

$$F < (\mu + B_0)(\gamma - \alpha B_0) \leq \frac{(\mu\alpha + \gamma)^2}{4\alpha}. \qquad (6)$$

To summarize, in order for the Galactic magnetic field to grow to its present strength, the flux F of magnetic monopoles must either obey bound (4) or bound (6). We also note that with $\alpha = 0$, V can have no stable fixed point (for B > 0). The physical significance of this result is that a flux a monopoles cannot (by itself) regulate the magnetic field strength in the Galaxy.

The flux limits derived above depend on the value of the seed magnetic field. For γ of order unity, a seed field larger than 10^{-20} G is required to produce the currently observed field strength [13,14]. The origin of the seed field is unknown, though several generation mechanisms have been proposed [15,16]. Based on these proposed mechanisms, we consider a range $10^{-20}\mathrm{G} \leq B_0 \leq 10^{-11}$ G. Our new monopole flux limit is shown as a function of mass in Fig. 2. For $m \leq 10^{17}$ GeV, our bound is tighter than the previous Parker bound. A simple analytic estimate of our flux limit is

$$\mathcal{F} \leq 1.2 \times 10^{-16}\left(\frac{m}{10^{17}\mathrm{GeV}}\right)\mathrm{cm}^{-2}\mathrm{s}^{-1}\mathrm{sr}^{-1} \qquad (7)$$

for $m \gtrsim 3 \times 10^{11}\mathrm{GeV}(B_0/10^{-11}\mathrm{G})$, and

$$\mathcal{F} \lesssim 3 \times 10^{-22}\mathrm{cm}^{-2}\mathrm{s}^{-1}\mathrm{sr}^{-1}(B_0/10^{-11}\mathrm{G}). \qquad (8)$$

for $m \lesssim 3 \times 10^{11}\mathrm{GeV}(\mathrm{B}_0/10^{-11}\mathrm{G})$.

Protogalactic Collapse: We now consider our second flux limit, obtained from considering the fact that some sort of seed magnetic field must survive during the formation of the Galaxy. During protogalactic collapse, Eq. (1) can be written as

$$\frac{dB}{dt} = \gamma_{\mathrm{coll}}\,B - \frac{gF}{1+\mu/B}. \tag{9}$$

Here, the amplification of the field is due to flux freezing, which produces an effective growth rate $\gamma_{\mathrm{coll}} \approx 2/\tau_{\mathrm{coll}}$, where τ_{coll} is the collapse time of the protogalaxy. Turbulent dissipation is not important for the small field values at this time. Since the protogalactic collapse time is expected to be $\sim 10^9$ yr [17], the growth rate $\gamma_{\mathrm{coll}} \sim 0.2$ in the dimensionless units defined previously. The dissipation term due to a flux of monopoles has the same form as before. However, the coherence length ℓ of the field is likely to be much larger during protogalactic collapse than today; this difference changes the value of μ for a given monopole mass, $\mu \propto \ell^{-1}$. Although the value is highly uncertain, we expect $\ell \sim$ 10–100 kpc.

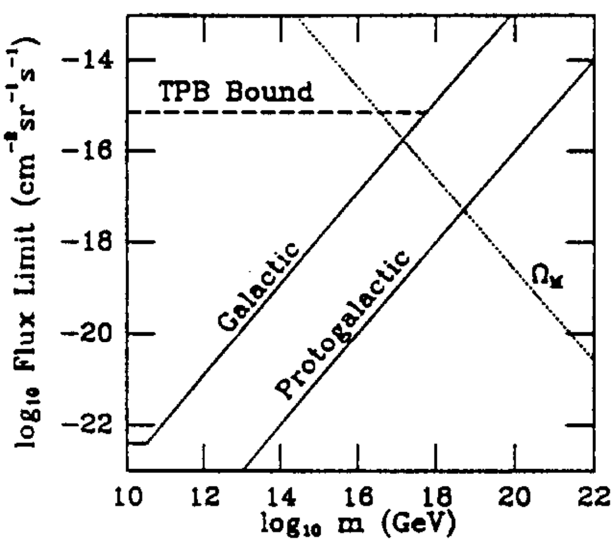

FIG. 2. Monopole flux limits as a function of the monopole mass m in GeV. The line labelled TPB Bound shows the modified Parker bound obtained in Ref. 8. The solid lines show the extended Parker bounds of this paper. The line labeled Ω_M represents the bound obtained by assuming monopoles are uniformly distributed throughout the universe but do not over close the universe. If the monopoles are clustered with galaxies, this closure bound becomes weaker by a factor of 10^5.

If the monopole flux $F < \mu\gamma_{\mathrm{coll}}$, then the field survives and grows for all initial values of the field strength. If, on the other hand, $F > \mu\gamma_{\mathrm{coll}}$, then only initial field strengths $B > B_C \equiv F/\gamma_{\mathrm{coll}} - \mu$ will survive and grow. Thus, in order for a protogalactic field of strength B_i to survive during collapse, the flux of monopoles *at that time* must obey one of the two bounds $F < \mu\gamma_{\mathrm{coll}}$, or, $F < (B_i+\mu)\gamma_{\mathrm{coll}}$. We note that $\mathcal{F} = nv/4\pi$ where n is the monopole number density, which scales with redshift as $(1+z)^3$. Thus, the flux of monopoles today is constrained to be smaller than the bound described above by a factor of $(1+z_{\mathrm{gal}})^3 \sim 10$, where z_{gal} is the redshift of galaxy formation. This estimate ($z_{\mathrm{gal}} \simeq 1.2$) is conservative and is applicable to normal galaxies. For $\gamma_{\mathrm{coll}} = 0.2$ and $\ell = 30$ kpc, we thus have roughly

$$\mathcal{F} \leq 10^{-19}\left(\frac{m}{10^{17}\mathrm{GeV}}\right)\mathrm{cm}^{-2}\mathrm{s}^{-1}\mathrm{sr}^{-1}. \tag{10}$$

It is evident from the above discussion that while the protogalactic collapse provides a tighter bound on the monopole flux than the galactic seed field bound, the uncertainties involved are more severe. Indeed, one could consider the implications of the survival of a seed field at earlier and earlier epochs, but with each additional "step" back, more uncertainties arise.

The net result of this paper is to place further restrictions on the allowed flux of magnetic monopoles. These limits are given as analytic formulae in Eqs. (4), (6), (7), (8) and (10). In order to display all of these bounds on a single graph, we must adopt particular choices for the parameters. For the bounds (4) and (6), we take a seed field strength $B_0 = 10^{-12}$ G. For the protogalactic bound (10), we take $(1+z_{\mathrm{gal}})^3 = 10$, $\gamma_{\mathrm{coll}} = 0.2$, $\ell = 30$ kpc and consider $B_i = 10^{-20}$ G. All other parameters are set to unity. The resulting bounds as a function of the monopole mass are shown in Fig. 2.

The bounds presented here are more stringent than previous Parker bounds for monopole masses below $\sim 10^{17}$GeV, and they have been obtained using conservative assumptions. *In principle*, Parker-type bounds can be evaded if monopoles participate in the maintenence of the galactic magnetic field through coherent oscillations [10,18]; in this circumstance the kinetic energy gained by monopoles is returned back to the field a half cycle later. However, it seems unlikely that monopole oscillations can maintain the necessary spatial and temporal coherence in the face of galactic inhomogeneities and their gravitational velocity dispersion [19]. Moreover, such scenarios cannot explain the present field strength through the growth of a very small seed field.

Figure 3 shows the current experimental situation where the most stringent experimental flux limits (90% C.L.) have been plotted vs. velocity. Indirect searches involving techniques such as etched nuclear tracks or catalysis that require assumptions other than the electromagnetic interaction of the monopole have been omitted. The combined limit for all searches based on magnetic induction has been obtained from Ref. [20]. The Baksan result [21] and the current MACRO result [22], are based on scintillation and together define the best limits to date for astrophysically interesting monopole velocities. The MACRO experiment [7] is just now becoming fully operational and will approach a sensitivity of $10^{-16}\mathrm{cm}^{-2}\mathrm{sr}^{-1}\mathrm{s}^{-1}$ in a little over five years.

As noted previously, "heavy" monopoles ($\mu \gg B$) should be moving at speeds of order $10^{-3}c$ within the Galaxy, while light monopoles ($\mu \ll B$) move significantly faster, having been accelerated by the galactic magnetic field. To compare our flux limits with the experimental results, one must specify the monopole mass. For monopole masses $< 10^{17}$ GeV the extended Parker bound is more restrictive than the closure bound (see Fig. 2) and thus rules out the possibility that monopoles much lighter than 10^{17} GeV provide closure density for the universe.

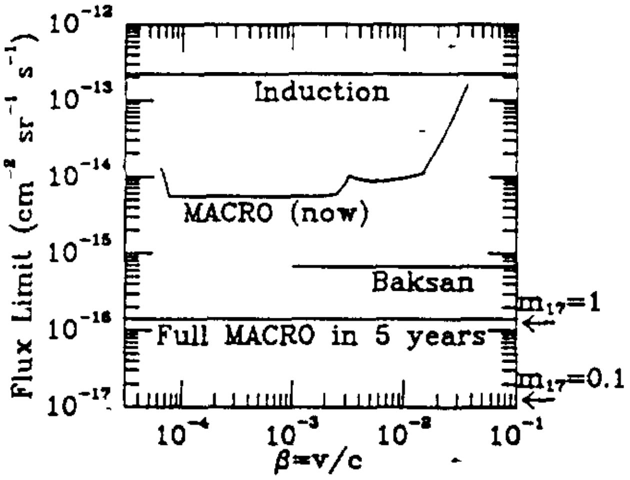

FIG. 3. Direct experimental monopole flux limits plotted as a function of the observed monopole velocity v for an isotropic flux (solid lines). The extended Parker flux limits have also been shown for monopole masses of 10^{17} GeV and 10^{16} GeV. The maximum monopole flux allowed by the extended Parker bound and the closure bound obtains for 10^{17} GeV.

We would like to thank A. Dolgov, P. Knezek, and E. Parker for useful discussions. GT is supported by DOE Contract DE-AC02-76ER01112; FCA and RW by NASA Grant No. NAGW–2802; MF by a Compton GRO Fellowship; KF by NSF Grant No. NSF-PHY-92-96020, a Sloan Foundation fellowship, and a Presidential Young Investigator award; MST by the DOE (at Fermilab and Chicago) and by NASA NAGW-2381 (at Fermilab).

[1] P.A.M. Dirac, *Phys. Rev.* 74, 817 (1948).

[2] G. 't Hooft, *Nuc. Phys.* B. 79, 276 (1974); A. Polyakov, *JETP Lett.*, 20, 194 (1974).

[3] For a review of magnetic monopoles and flux limits see, e.g., J. Preskill, *Ann. Rev. Nucl. Part. Sci.* 34, 461 (1984).

[4] A.H. Guth, *Phys. Rev. D* 23, 347 (1981).

[5] B. Cabrera, *Phys. Rev. Lett.* 48, 1378 (1982); B. Cabrera et al *Phys. Rev. Lett.* 51, 1933 (1983).

[6] S.P. Ahlen and G. Tarlé, *Phys. Rev. Lett.* D27, 688 (1983); S.P. Ahlen et al, *Phys. Rev. D.* 36, 311(RC) (1987).

[7] The MACRO Collaboration, *Nucl. Instr. and Meth.* A264, 18 (1988).

[8] J.A. Frieman, K. Freese and M.S. Turner, *Ap. J.* 335, 844 (1988); E.W. Kolb, S. Colgate and J.A. Harvey, *Phys. Rev. Lett.* 49, 1373 (1982); S. Dimopoulos, J. Preskill and F. Wilczek, *Phys. Lett.* 119B, 320 (1982); E.W. Kolb and M.S. Turner, *Ap. J.* 286, 702 (1984); K. Freese, M.S. Turner and D.N. Schramm, *Phys. Rev. Lett.* 51, 1625 (1983); K. Freese, *Ap. J.* 286, 216 (1984).

[9] E.N. Parker, *Ap. J.* 160, 383 (1970).

[10] M.S. Turner, E.N. Parker and T. Bogdan, *Phys. Rev.* D 26, 1296 (1982).

[11] E.N. Parker, *Ap. J.* 163, 255 (1971).

[12] E.N. Parker, *Ap. J.* 163, 279 (1971).

[13] Ya.B. Zel'dovich, A.A. Ruzmaikin, and D.D. Sokoloff, *Magnetic Fields in Astrophysics* (Gordon and Breach, New York, 1983).

[14] E.N. Parker, *Cosmical Magnetic Fields* (Clarendeon Press, Oxford, 1979).

[15] E.R. Harrison, *Mon. Not. R. Astron. Soc.* 147, 279 (1970); A. Vilenkin and D.A. Leahy, *Ap. J.* 254, 77 (1982); C.J. Hogan, *Phys. Rev. Lett.* 51, 1488 (1983); M.S. Turner and L.M. Widrow, *Phys. Rev.* D 51, 2743 (1988); J.M. Quashnock, A. Loeb, and D.N. Spergel, *Astrophys. J.* 254, 77 (1988); T. Vachaspati, *Phys. Lett.* B 265, 258 (1991); B. Ratra, *Ap. J.*, 391, L1 (1992); A.D. Dolgov and J. Silk, *Phys. Rev.* D, in press (1992); A.D. Dolgov and S.H. Rhie, preprint (1992); W.D. Garretson, G.B. Field, and S.M. Carroll, preprint (1992).

[16] E.T. Vishniac, preprint (1992).

[17] J. Silk, *Nature* 351, 191 (1991).

[18] E. Salpeter, S. Shapiro, and I. Wasserman, *Phys. Rev. Lett.* 49, 1114 (1982); J. Arons and R. Blandford, *Phys. Rev. Lett.* 50, 544 (1983).

[19] E. N. Parker, *Ap. J.* 321, 349 (1987).

[20] M.E. Huber, *Phys. Rev. Lett.* 64, 835 (1990); S. Bermon, *Phys. Rev. Lett.* 64, 839 (1990).

[21] E.N. Alexeyev et al, *in Proc. of the 21st Int. Cosmic Ray Conf.*, Adelaide, Australia, 10, 83 (1990).

[22] J. Hong, Ph.D. Thesis, Caltech (1993).